Mathematics and Origami
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Traditional Approach to Origami Design:

Fold paper symmetrically until it looks like something.

- Somewhat hit and miss
- Constrained by symmetries of original square
Contemporary Approach to Origami Design:

Plan where different parts of the paper should end up.

- New techniques deliberately place the paper where it is needed.
- Constraint of using square paper with no cuts can be met without allowing symmetries to dictate the design.
The East and West of Origami History
Eastern Tradition (China, Japan, Korea):

- Used many different shapes of starting paper
- Used cuts freely
- Commonly used paper specially colored and designed to suit the subject
- Used lots of judgement folds (also called RAT folds for “Right About There”)
- Designs featured many 22.5 degree angle folds (half of 45 degrees)
- Pursued by skilled and wealthy adults
- Considered an expression of the sincerity of the folder
- Included many simple animals and flowers
Western Tradition (Northern Africa, Spain, Europe):

- Usually used square or rectangular paper
- Seldom used cuts
- Rarely used paper specifically designed for the folding project
- Favored folds that could be constructed without guessing
- Designs featured 45 and 90 degree angle folds
- Pursued as a mathematical exploration, folk craft, or childhood passtime
- Original Arabic tradition included geometric designs and boxes
- European tradition included paper toys and action models
Contemporary approach blends technical and aesthetic ideas from both traditions and extends them

- Emphasis on realism
- Rules and constraints – cuts, painting, shape of paper
- Animals, flowers, masks, toys, boxes, tessellations, geometric objects
- Origami as a Visual Art Form
Classic Bases and Origami Design

- Theory developed in the early 1900s
- Folders noticed common initial folding sequences in traditional designs
- A base forms the skeleton for a fold
- Different bases provide different numbers of points in different arrangements
- In mid 1900s, folders typically made new designs by choosing the optimal classic base for the task
- This was a useful approach, but was artistically limiting
- Awareness of classic bases helped designers realize they needed to break away from well-trodden artistic ground
Folding Beyond the Classic Bases

- Extending classic bases by blintzing
- Point splitting
- Base-free folding
- Stumbling on new bases by good fortune and intuition
- Cross-breeding classic bases
- Grafting
Tessellations
Modular Origami
Origami Constructions

- Dividing paper into an arbitrary number of equal strips
- Dividing angles evenly
- Huzita’s Origami Axioms
Analysis of Paper Folding Limits

- Flat Folding
- Color Changes
- Folding curved surfaces
Folding Paper In Half

• Many experts thought it was impossible to fold a piece of paper in half more than 5 or 6 times.
• High school student Britney Gallivan showed that the estimate they used was wrong.
• She proved it is possible to fold paper in half as many times as you like if the paper is long enough.
• She developed new equations that tell how long the paper needs to be given its thickness and the number of folds you want.
Folding Long Points

- Half a century ago, many folders believed it was impossible to fold something with more than four long appendages.
- Folders have since found many ways to make arbitrarily many long points.
Tree Maker Algorithm

Robert Lang has developed a computer algorithm that can create a crease pattern for any stick figure outline.

This tool has enabled artists to greatly increase the complexity of folded figures.
The One-Cut Theorem

Erik Demaine proved that there is always a way to fold a piece of paper so that any plane graph of line segments can be sliced with a single straight cut.
Applications of
Mathematical Origami
A Few Active Research Directions

- Differential geometry of straight and curved folds
- Extend the origami axioms to include simultaneous alignments
- Computability and decidability questions in origami (mountain and valley assignments, for example)
• Algorithms for folding classes of mazes, tessellations, fractals, stick figures, words, et cetera

• Colorings and origami designs

• Creative limitations resulting from restrictions to reduced sets of allowed folds (folding allowed only in certain directions, only certain folding motions permitted, et cetera)
• Folding materials with non-flat curvature

• Origami in higher dimensions

• Computer programs that can create a folding pattern and directions for making any desired form

• New engineering applications based on origami mechanisms (tessellated nanotechnology materials, studies of optimal crumpling patterns, creating self-folding materials, training bacteria to fold materials)